

# Extracting Optimal Performance from Dynamic Time Warping

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## ABSTRACT

Dynamic Time Warping (DTW) is a distance measure that compares two time series after optimally aligning them. DTW is being used for decades in thousands of academic and industrial projects despite the very expensive computational complexity,  $O(n^2)$ . These applications include data mining, image processing, signal processing, robotics and computer graphics among many others. In spite of all this research effort, there are many myths and misunderstanding about DTW in the literature, for example “it is too slow to be useful” or “the warping window size does not matter much.”

In this tutorial, we correct these misunderstandings and we summarize the research efforts in optimizing both the efficiency and effectiveness of both the basic DTW algorithm, and of the higher-level algorithms that exploit DTW such as similarity search, clustering and classification. We will discuss variants of DTW such as constrained DTW, multi-dimensional DTW and asynchronous DTW, and optimization techniques such as lower bounding, early abandoning, run-length encoding, bounded approximation and hardware optimization.

We will discuss a multitude of application areas including physiological monitoring, social media mining, activity recognition and animal sound processing. The optimization techniques are generalizable to other domains on various data types and problems.

## 1. TOPIC OVERVIEW

With the advent of novel sensing techniques, time series data are ubiquitous. Large volumes of time series data are routinely created in scientific, industrial, entertainment, medical and biological domains. Examples include physiological recordings, space telemetry, performance counters, accelerometer signals, oceanographic signals, acoustics signals, quantified self, financial indices, etc.

Dynamic Time Warping (DTW) is a distance measure for time series data that is invariant to distortions in time. DTW has been used for decades in many application areas in-

cluding speech processing, physiological monitoring, human activity recognition and financial engineering. Although time warping applications are ubiquitous, the computational complexity of DTW  $O(n^2)$  is a major barrier to adoption. At least two hundred research papers have been published that focus on speeding up DTW algorithm and its applications to higher-level algorithms. The current fastest DTW algorithm can search a trillion subsequences in a day, or monitor streams in real-time [13], yet we argue that the full power of DTW remains underexploited.

In this tutorial, we focus on two key issues:

- How to do DTW efficiently
- How to do DTW effectively

**Efficient:** We will discuss many optimization techniques that have accelerated DTW algorithm and its usefulness in similarity search (in archived [6] and streaming data [16][13]), classification [3] and clustering (exact [18] and approximate [22]). The optimization techniques include early abandoning [13], lower bounding [8][9], bounded approximation [22], compressed comparison and hardware acceleration [17] among many others. These techniques have been successfully used in many domains and are generalizable to many data types and problems commonly appearing in other domains. Thus attendees working on different data types (strings, graphs, images etc) will still come away with useful tools and ideas.

**Effective:** Many published applications of DTW do not achieve the quality of results that are possible, because the users make suboptimal choices. For example, not normalizing the data properly, not using the correct “warping window” parameter, not generalizing DTW to multidimensional data correctly, enforcing the “endpoint” constraint where it should not be used etc. We will explain these issues, and show how to squeeze the optimal performance out of DTW.

The tutorial will be illustrated with numerous real world examples created just for this tutorial, including case studies in physiology [20], social media, activity recognition [2], speech processing [11] and image processing [14]. We will end the tutorial with a set of open problems that are challenging and have potential to significantly advance the knowledge front. We will also discuss some closed problems that are either solved or do not exist anymore. The key technical challenges and available sources of data will be discussed.

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## 2. COVERAGE AND RELATED WORK

The tutorial covers selected research published in top conferences (e.g. VLDB [6][8][19], SIGKDD [3][13][1][7], ICDE [9][16][21][2], ICDM [12][17][10] and SDM [4][18][22]). We will present a comparative picture of all these methods from different perspectives including efficiency, accuracy, interpretability, resource requirement and so on. In addition, there is considerable research work on time series that appears outside the mainstream data mining conferences; for example, much interesting work on time series is done in Pervasive technologies [5][20], computer vision (CVPR) [14] and speech processing (Tran. Acoustics) [11][15]. We have also made extensive efforts to summarize such work in our comparative study.

## 3. PRESENTER BIOS

**Dr. Mueen** is an Assistant Professor at Computer Science in University of New Mexico. He has won the runner-up of SIGKDD doctoral dissertation contest in 2012 and a best paper award in the same conference. His research work has been published in top conferences including SIGMOD, KDD and ICDM. He has recently given well received tutorials in SDM and ICDM conferences on repeated pattern mining from time series data.

**Dr. Keogh** is a prolific author in data mining, for example he is a top-ten most prolific author in all three of the top ranked data mining conferences, SIGKDD, ICDM and SDM. He has won best paper awards at ICDM, SIGKDD and SIGMOD. His H-index of 68 reflects the influence of time series representations and algorithms. He has given well-received tutorials at SIGKDD (three times), ICDM (four times), VLDB, SDM, and CIKM.

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